Influence of the Wall on Transient Conduction into Packed Media

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Surfaces immersed in moving packed beds and fluidized beds frequently experience a "renewal" type of contact with packed media. If there is heat transfer during surface renewal, it can be modeled as a transient conduction process provided the contribution due to thermal radiation is negligible. There exist two distinct approaches (Mickley and Fairbanks, 1955; Botterill and Williams, 1963) for the prediction of the heat transfer coefficient. Many investigators have subsequently suggested modifications to the basic models cited above so as to achieve good agreement with experimental data (Saxena and Gabor, 1981; Chandran and Chen, 1985).

A review of the literature indicated that the model developed by Kubie and Broughton (1975) exhibits satisfactory agreement with experimental data without the use of empirical parameters. However, the model requires a numerical solution and that detracts it from use for design purposes. From an application standpoint, it is desirable to develop a simple closed-form relation for the heat transfer coefficient; such a relation is derived in this note.

ANALYSIS

Consider a disperse system comprising a gas-solid medium with a close-packed matrix of solid particles and stationary gas that comes into contact with a surface at a different temperature for a certain duration. For the case of negligible radiant contribution, heat transfer occurs by unsteady state conduction. In formulating the problem, it is important to (i) account for the influence of the surface or wall on the local packing, and (ii) recognize the heterogeneity of the two-phase medium. The property boundary layer concept introduced by Kubie and Broughton (1975) is used here to account for the wall effect. This translates into the voidage variation shown in Figure 1. The heterogeneous nature of the disperse system, however, is more difficult to deal with. Hence as a first approximation, the stochastic behavior of multiple particles is treated as pseudo-homogeneous in this work.

For constant wall temperature, with the usual assumptions, the governing equations for transient conduction into the disperse system can be written in normalized form as:

$$\frac{\partial \phi}{\partial Fo_i} = \frac{k_g}{k_D z(3-2z)} \frac{\partial}{\partial z}\left[\left\{1 + z(3-2z)\left(\frac{k_D}{k_g} - 1\right)\right\}\frac{\partial \phi}{\partial z}\right] \quad \text{for } z \le 1 \tag{1}$$

$$\frac{\partial \eta}{\partial Fo_i} = \frac{\partial^2 \eta}{\partial z^2} \quad \text{for } z \ge 1 \tag{2}$$

subject to

$$\phi(z,0) = 0 \text{ for } z \le 1 \tag{3}$$

$$\eta(z,0) = 0 \text{ for } z \ge 1 \tag{4}$$

$$\phi(0,Fo_i) = (T_W - T_B) = \text{constant} \tag{5}$$

$$\lim_{Z \to \infty} \eta(z,Fo_i) = 0 \tag{6}$$

$$\phi(1,Fo_i) = \eta(1,Fo_i) \tag{7}$$

$$k(z)\left.\frac{\partial \phi}{\partial z}\right|_{z=1,Fo_i} = k_D \left.\frac{\partial \eta}{\partial z}\right|_{z=1,Fo_i} \tag{8}$$

where

$$z = x/\bar{d}_p, \quad \phi = T_1 - T_B, \quad \eta = T_2 - T_B,$$
$$Fo_i = (k_D t)/[(\rho c_p)_D \bar{d}_p^2]$$

$$C_{pD} = (\rho c_p)_D \simeq \rho_s c_{ps}(1 - \alpha_D) \text{ for } \frac{\rho_g c_{pg}}{\rho_s c_{ps}} \ll 1 \tag{9}$$

Equations 1 and 2 were obtained by using the relation proposed by Kubie and Broughton (1975) for voidage variation in the vicinity of a constraining surface (Figure 1):

$$\begin{aligned} \alpha &= 1 - 3\,(1 - \alpha_D)\left(z - \frac{2}{3}z^2\right) && \text{for } z \le 1 \\ &= \alpha_D && \text{for } z \ge 1 \end{aligned} \tag{10}$$

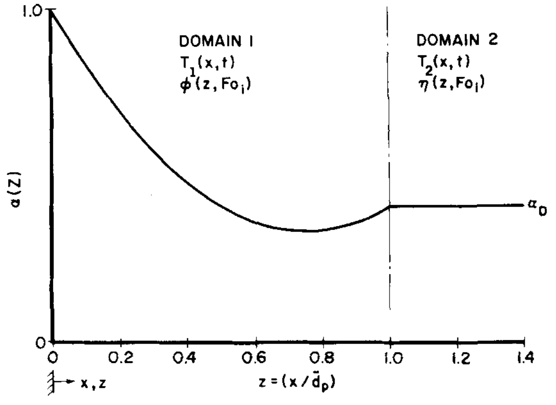

Figure 1. Voidage variation near the wall.

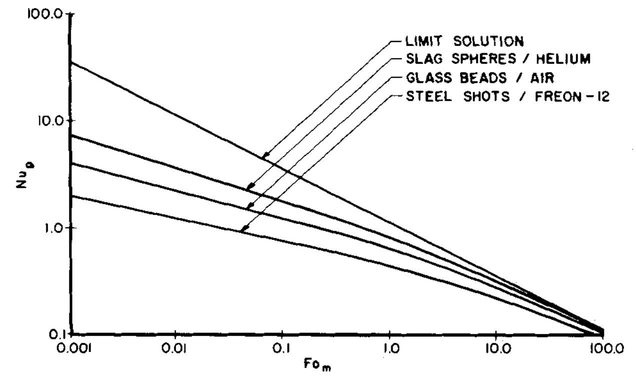

Figure 2. Variation of transient conduction solution with Fourier modulus and conductivity ratio.

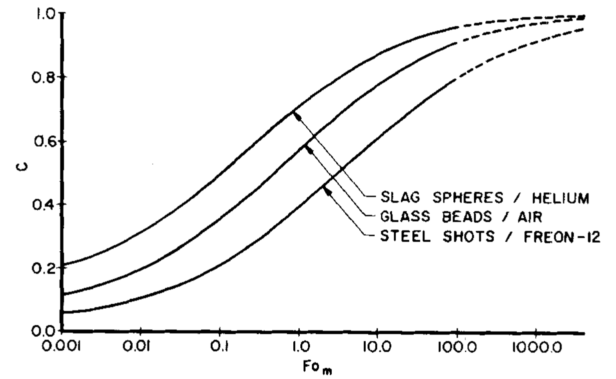

Figure 3. Variation of effective correction factor with Fourier modulus and conductivity ratio.

NUMERICAL SOLUTION

The governing equations (Eqs. 1–8) were solved by a numerical technique (Chandran, 1980). For a fixed value of the conductivity ratio (k_D/k_g), numerical solutions were obtained as a function of the instantaneous Fourier number (Fo_t). From these results, the corresponding values of the instantaneous heat transfer coefficient (h_{pi}) were computed; these were then integrated to determine the values of the time-averaged heat transfer coefficient (h_p) and the time-mean Nusselt number ($Nu_p = h_p\bar{d}_p/k_D$) for time-mean Fourier modulus ($Fo_m = k_D\,\bar{\theta}_D/C_{pD}\bar{d}_p^2$) ranging from 0.001 to 100.0. Here, the variation of the time-mean Fourier modulus (Fo_m) corresponds to a change in the disperse system mean residence time ($\bar{\theta}_D$). The foregoing procedure was repeated for seven different values of the conductivity ratio between 1.4 and 34.7. Figure 2 shows the variation of Nusselt number (Nu_p) with Fourier modulus (Fo_m) for three different solid-fluid combinations—slag spheres/helium ($k_D/k_g = 2.21$), glass beads/air ($k_D/k_g = 6.03$) and steel shots/freon-12 ($k_D/k_g = 20.0$). The limit solution obtained from "the packet theory" (Mickley and Fairbanks, 1955; Mickley et al., 1961) is also included. It is seen that the numerical solutions approach the limit solution for large values of the Fourier modulus ($Fo_m > 100$), but depart appreciably from the asymptotic solution at small values of Fo_m (<1). Also the extent of deviation increases with the conductivity ratio (k_D/k_g).

DESIGN CORRELATION

The numerical solution presented above is not in a readily useable form for design purposes. Hence there is a need for developing an algebraic relation for the time-average heat transfer coefficient. This is accomplished here by modifying the packet theory limit solution and then matching the modified solution with the numerical solution obtained in the previous section.

Consider the case where the disperse system remains in contact with a wall for a mean residence time of $\bar{\theta}_D$. From packet theory (Mickley and Fairbanks, 1955; Mickley et al., 1961; Ozkaynak and Chen, 1980), the time-mean local heat transfer coefficient (h_s) for constant wall temperature boundary condition is

$$h_s = 2\sqrt{\frac{k_e C_{pe}}{\pi\bar{\theta}_D}} \tag{11}$$

The physical properties (k_e, C_{pe}) should correspond to *effective* values for the disperse system in the region adjacent to the heat transfer surface. If the heat wave penetrates several particle diameters deep into the medium, these properties can be assigned the "bulk" values for the disperse system. Hence, for long residence times ($Fo_m > 100$):

$$k_e \simeq k_D \text{ and } c_{pe} \simeq C_{pD}$$

For short residence times where the Fourier modulus is small [$Fo_m < 1$], the effective zone for transient conduction in the packed media is of the order of a fraction of a particle-diameter in depth. Since it is known that in the first particle layer the local void fraction is greater than α_D (Korolev et al., 1971; Kubie and Broughton, 1975; Pillai, 1977; Ozkaynak and Chen, 1980), the effective values k_e and C_{pe} ought to be less than the respective bulk values k_D and C_{pD}. It is suggested that two correction factors c_1 and c_2 be used to accountt for this effect:

$$k_e = c_1 \cdot k_D$$
$$C_{pe} = c_2 \cdot C_{pD} \tag{12}$$

where c_1 and c_2 are expected to be bounded in magnitude between 0 and 1. The values of these two factors would depend upon the parameters that influence the "penetration depth." Thus, c_1 and c_2 are likely to vary with the Fourier modulus (Fo_m) and the conductivity ratio (k_D/k_g); based on physical considerations, the correction factors ought to have the following limits:

$$\lim_{Fo_m \to 0} c_i = 0 \qquad \text{for } i = 1,2 \qquad \lim_{Fo_m \to \infty} c_i = 1 \tag{13}$$

By substituting relations (Eq. 12) in Eq. 11 and defining an effective correction factor ($C \equiv \sqrt{c_1 \cdot c_2}$), the following expression is obtained

$$h_s = 2C\sqrt{\frac{k_D C_{pD}}{\pi\bar{\theta}_D}} \tag{14}$$

Thus, the heat transfer coefficient due to surface renewal (h_s) could be evaluated by multiplying the packet theory limit solution with a correction factor (C).

It is now necessary to determine the functional dependence of C on Fo_m and (k_D/k_g). This is accomplished by matching the modified solution given by equation (14) with the numerical solution (h_p) obtained earlier. The correction factor (C), determined thus, exhibits the variation shown in Figure 3. It is seen that C increases with increasing Fo_m and approaches 0 and 1 in the limits of zero and infinite Fo_m respectively. Also, for a given Fourier number a larger value of C is obtained for a smaller conductivity ratio. By regression analysis, the ensuing correlation was obtained for C:

$$C = \exp\left[-\frac{a_1}{Fo_m^{a_2 + a_3 ln Fo_m}}\right] \tag{15}$$

with

$$a_1 = 0.213 + 0.117w + 0.041w^2$$
$$a_2 = 0.398 - 0.049w$$
$$a_3 = 0.022 - 0.003w$$
$$w = ln(k_D/k_g)$$

Equation 14 with C as given by Eq. 15 in effect represents the numerical solution (agreement within 2%) and serves as a convenient design correlation.

NOTATION

a_1,a_2,a_3 = coefficients in Eq. 15
C = effective correction factor, Eq. 14
C_p = volumetric specific heat, ρc_p
c_1,c_2 = property correction factors, Eq. 12
c_p = specific heat
D = tube diameter
$\overline{d}_p$ = mean particle diameter
Fo_i = instantaneous Fourier number, $(k_D t)/[(\rho c_p)_D \overline{d}_p^2]$
Fo_m = time-mean Fourier modulus $(k_D \overline{\theta}_D)/(C_{pD} \overline{d}_p^2)$
h_{pi} = instantaneous heat transfer coefficient during transient conduction (numerical solution)
h_p = time-averaged heat transfer coefficient for disperse system residence time of $\overline{\theta}_D$ (numerical solution)
h_s = heat transfer coefficient due to surface renewal of a gas-solid medium with stagnant fluid (modified solution)
k = thermal conductivity
Nu_p = time-mean Nusselt number, $(h_p \overline{d}_p)/k_D$ (numerical solution)
T = temperature
T_1 = disperse system temperature in domain 1, Figure 1
T_2 = disperse system temperature in domain 2, Figure 1
t = time
w = natural log of conductivity ratio, $ln(k_D/k_g)$
x = distance from the wall, Figure 1
z = dimensionless distance from the wall, $x/\overline{d}_p$

Greek Letters

α = void fraction
η = excess temperature, $(T_2 - T_B)$
$\overline{\theta}_D$ = disperse system mean residence time
ρ = density
ϕ = excess temperature, $(T_1 - T_B)$

Subscripts

B = bed, (away from the wall)
D = disperse system
e = effective value
g = gas
s = solid/particle
w = immersed surface/wall

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Hydrodeoxygenation of 1-Naphthol Catalyzed by Sulfided Ni-Mo/γ-Al_2O_3: Reaction Network

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Liquids derived from coal and shale have high concentrations of organooxygen compounds that react with hydrogen under conditions of catalytic hydroprocessing to give hydrocarbons and water. These reactions are poorly characterized, and an understanding of the reaction networks and kinetics is expected to be of value in process design and modeling. Most of the few publications concerned with hydrodeoxygenation (HDO) reactions are concerned with (substituted) phenols and dibenzofuran. Hydrodeoxygenation of dibenzofuran in the presence of industrial hydro-

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